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# The Exclusive Decay $B \rightarrow \rho e \nu$ Beyond Model Calculations

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## Abstract

Due to its comparatively theoretical “simplicity”, the decay channel  $B \rightarrow \rho e \nu$  offers one of the best possibilities to determine the CKM matrix element  $|V_{ub}|$  accurately. I present a new calculation of the relevant hadronic form factors from light-cone sum rules. I also review the results from lattice calculations and find that they agree with the results from light-cone sum rules where comparison is possible.

This paper relies on work done in collaboration with V.M. Braun.

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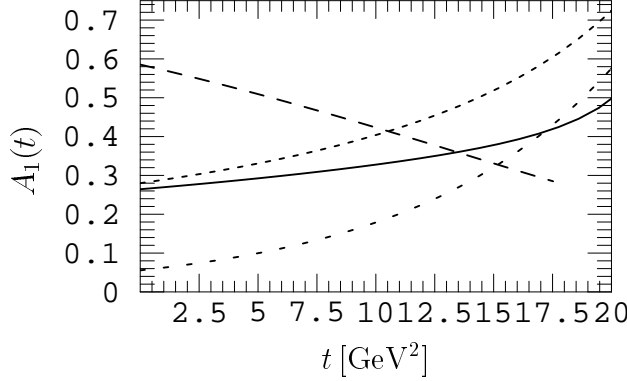


Figure 1: A sample of predictions for  $A_1(t)$  from quark models and QCD sum rules. Solid line: light-cone sum rules [2]; long dashes: three-point sum rules [1]; short dashes with short spaces: BWS model [3]; short dashes with long spaces: ISGW model [4].

With the increasing statistics of experimental data from CLEO, the determination of  $|V_{ub}|$  from the decay channel  $B \rightarrow \rho e \nu$  becomes more and more feasible. Experimentally,  $b \rightarrow u$  transitions are visible only above the kinematical threshold for charm production, i.e. for electron energies  $E_e > 2.3 \text{ GeV}$  in semileptonic decays. The theoretical description of inclusive decays being exceedingly difficult in that region, it seems more promising to look into the exclusive channels. Here it is naturally the  $B \rightarrow \pi$  and  $B \rightarrow \rho$  transitions that, if at all, are tractable theoretically, and of these the latter is the most prospective one, as it is expected to be strongly peaked in the observable region, cf. Fig. 11 in Ref. [1]. It is thus timely to review the existing theoretical predictions for these decays.

Let us begin with definig the relevant hadronic matrix element for  $B \rightarrow \rho$  transitions:

$$\begin{aligned} \langle \rho, \lambda | (V - A)_\mu | B \rangle = & -i(m_B + m_\rho)A_1(t)\epsilon_\mu^{*(\lambda)} + \frac{iA_2(t)}{m_B + m_\rho}(\epsilon_\mu^{*(\lambda)}p_B)(p_B + p_\rho)_\mu \\ & + \frac{iA_3(t)}{m_B + m_\rho}(\epsilon_\mu^{*(\lambda)}p_B)(p_B - p_\rho)_\mu + \frac{2V(t)}{m_B + m_\rho}\epsilon_\mu^{\nu\rho\sigma}\epsilon_\nu^{*(\lambda)}p_{B\rho}p_{\rho\sigma}, \end{aligned} \quad (1)$$

where the four form factors  $A_{1,2,3}$  and  $V$  depend on the momentum transfer  $t = (p_B - p_\rho)^2$  to the leptons; in the limit of vanishing lepton mass  $A_3$  does not contribute to the semileptonic decay rate and will not be considered in this note.  $\lambda$  is the polarization of the  $\rho$  meson.

Fig. 1 shows a sample of predictions for the form factor  $A_1$ , the situation with the others is similar. It is obvious that the spread of predictions prevents any reliable prediction of the decay rate. The problem is essentially the large range of physical values of  $t$ ,  $0 \leq t \leq 20.3 \text{ GeV}^2$ , which are all relevant for the observable electron spectrum. The lack of predictiveness of the dynamical properties of form factors is a common feature of most quark models [3, 4, 5] and forces them to rely on ad hoc assumptions on the  $t$ -dependence like the pole dominance hypothesis. However, for B decays an accurate *prediction* of the  $t$ -dependence is absolutely mandatory. At present there are only two methods that can claim some right to make such predictions founded in QCD, that is the QCD sum rules method on the one hand and lattice calculations on the other hand.

Let us shortly review the status of the latter ones. Due to the restricted size of presently tractable lattices, typical masses of simulated hadrons are around 1.5 to 3 GeV. Extraction of B physics from the lattice thus requires the extrapolation of the results obtained for small quark masses to the B scale. For form factors the situation is even more complicated due to the presence of a second potentially large scale, the momentum transfer  $t$ . The extrapolation in the heavy mass  $m_H$  can be done either “naïvely”, by fitting to the ansatz  $F = A + B/m_H$ , where F stands for a generic form factor [6, 7] (the extrapolation is done at fixed three-momentum of the final state meson in the rest system of the decaying particle, i.e.  $t$  scales with the heavy mass, too), or by using the guidance provided by heavy quark effective theory, according to which near maximum  $t$  the form factors behave apart from logarithmic corrections in the heavy mass as [8]:

$$A_1(t_{\max}) \sim 1/\sqrt{m_H}, \quad A_2(t_{\max}) \sim V(t_{\max}) \sim \sqrt{m_H}. \quad (2)$$

The quantity constant in the heavy quark limit, i.e.  $F m_H^{\pm 1/2}$ , is then fitted by a polynomial in the inverse heavy mass, cf. [7, 9]. Working near  $t_{\max}$  poses however certain problems: first it is not clear in which range of  $t$  the above scaling laws remain valid. Second, one is clearly interested in the form factors in the full range of  $t$ , or, as for  $B \rightarrow K^* \gamma$ , only in the value at  $t = 0$ . Thus, without knowledge of the functional  $t$ -dependence of the form factors the extrapolation to smaller values of  $t$  becomes model-dependent<sup>1</sup>. For  $B \rightarrow \pi$  transitions it was attempted to restrict the functional  $t$ -dependence, given some values at large  $t$ , from unitarity arguments [10], but to my knowledge no similar bounds for the  $B \rightarrow \rho$  form factors exist to date.

Another possibility advocated in Refs. [9, 11] is to fix  $t = 0$  and then to extrapolate the data in the heavy mass. As stressed in [11], this method avoids the model-dependence from extrapolating down to  $t = 0$ , but it introduces another one, namely the leading behaviour of the form factors at  $t = 0$  in the heavy mass, which cannot be obtained from heavy quark effective theory. Ref. [9] fits to  $A_1(0) \sim m_H^{-3/2}$  (plus  $1/m_H$  corrections), which relies on the scaling law (2) and an assumed monopole pole behaviour of  $A_1(t)$ . Ref. [11] tries three different fits with  $F(0) \sim m_H^{-1/2, 0, 1/2}$ . Unfortunately, the data available to date do not allow to distinguish between different powers of  $m_H$ , although the extrapolated values are rather sensitive to it. In view of this difficulty, I would like to draw the lattice community’s attention to the fact that the leading behaviour of form factors at  $t = 0$  in the heavy quark limit *can* be extracted from QCD and turns out to be a decrease with  $m_H^{3/2}$ . This behaviour follows from the asymptotic form of the leading twist  $\rho$  meson light-cone wave functions in QCD. A comprehensive review on light-cone wave functions can be found in Ref. [12], whereas the heavy quark mass limit of form factors at  $t = 0$  was elaborated on in Refs. [13, 14, 2]. I hope that in future calculations this behaviour will be taken into account and lattice simulations will yield form factors both at  $t = 0$  and large  $t$ , which would thus allow to replace the hitherto used *extrapolations* in  $t$  by *interpolations*.

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<sup>1</sup>Actually most collaborations use a monopole behaviour, cf. [6, 7, 9], with which the available data at different (large)  $t$  are consistent, without, however, being conclusive.

Let me now turn to QCD sum rules. For the calculation of heavy-to-light transition form factors there exist actually two types of them. The more “traditional” ones are the “three-point sum rules” based on the classical approach of Shifman, Vainshtein and Zakharov [15] and rely on the expansion of a three-point correlation function in terms of local operators, whose vacuum matrix elements, the condensates, reflect the complicated nature of the QCD vacuum and, following Ref. [15], are used to describe non-perturbative corrections to the perturbative three-point functions. This method was applied to  $B \rightarrow \rho$  transitions in Refs. [16, 1].

A modification of that classical approach are the “light-cone sum rules” [17, 13, 18] that combine the QCD sum rule technique with an expansion in terms of (non-local) operators of definite *twist* (instead of dimension), whose matrix elements over the vacuum and the light hadron are described by light-cone wave functions. Predictions for  $B \rightarrow \rho$  form factors from this approach were given in Refs. [14, 2].

As discussed in Ref. [2], reliable predictions of form factors from three-point sum rules are restricted to large values of  $t$ , but fail for small  $t$ , i.e.  $t \sim O(m_b)$ . At small  $t$  the condensate contributions to these sum rules diverge in the heavy quark limit (for instance  $A_1(0)$  blows up as  $m_b^{1/2}$ ), which is in contradiction to what one would expect intuitively and also different from the behaviour of the perturbative contributions to the sum rules. Technically, three-point sum rules fail to describe large transverse distances in the hadronization of the  $\rho$  meson, which, however, are essential to one of the two dominant hadronization processes, the Feynman mechanism, cf. [13, 2]. These contributions are taken into account correctly in the light-cone sum rules approach, whose central objects are matrix elements of non-local operators that are expressed in terms of light-cone wave functions. For the  $\rho$  meson there are four wave functions up to twist three, which are of the generic type

$$\langle 0 | \bar{u}(x) \Gamma d(0) | \rho(p) \rangle |_{x^2=0} \sim \int_0^1 du e^{-iupx} \Phi(u). \quad (3)$$

Here  $\Gamma$  is some Dirac structure and  $u$  is the momentum fraction of the  $\rho$  carried by the quark. The wave function  $\Phi$  can be interpreted as the probability amplitude to find the  $\rho$  in a state with minimum number of Fock constituents and at small transverse separation. The twist two wave functions of the  $\rho$  were reexamined recently in Ref. [19].

The results for the form factors both from light-cone sum rules and lattice calculations<sup>2</sup> are shown in Fig. 2. The agreement of light-cone sum rules and lattice results within the error-bars is striking, in particular for  $A_1$ , which dominates the decay rate at large  $t$ . Comparing with the pole-dominance hypothesis, I find that  $A_1$  increases, but less than a monopole,  $A_2$  is comparable with a monopole, whereas  $V$  increases nearly as a dipole. As for the errors, there exists a very sophisticated error-analysis from lattice, whereas a corresponding analysis for the light-cone sum rule results it is not that easy. Neglecting possible systematic errors of the method, I obtain from varying the input parameters within

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<sup>2</sup>I only give the results from the UKQCD collaboration [9], which are the only ones that do not rely on the pole dominance hypothesis.

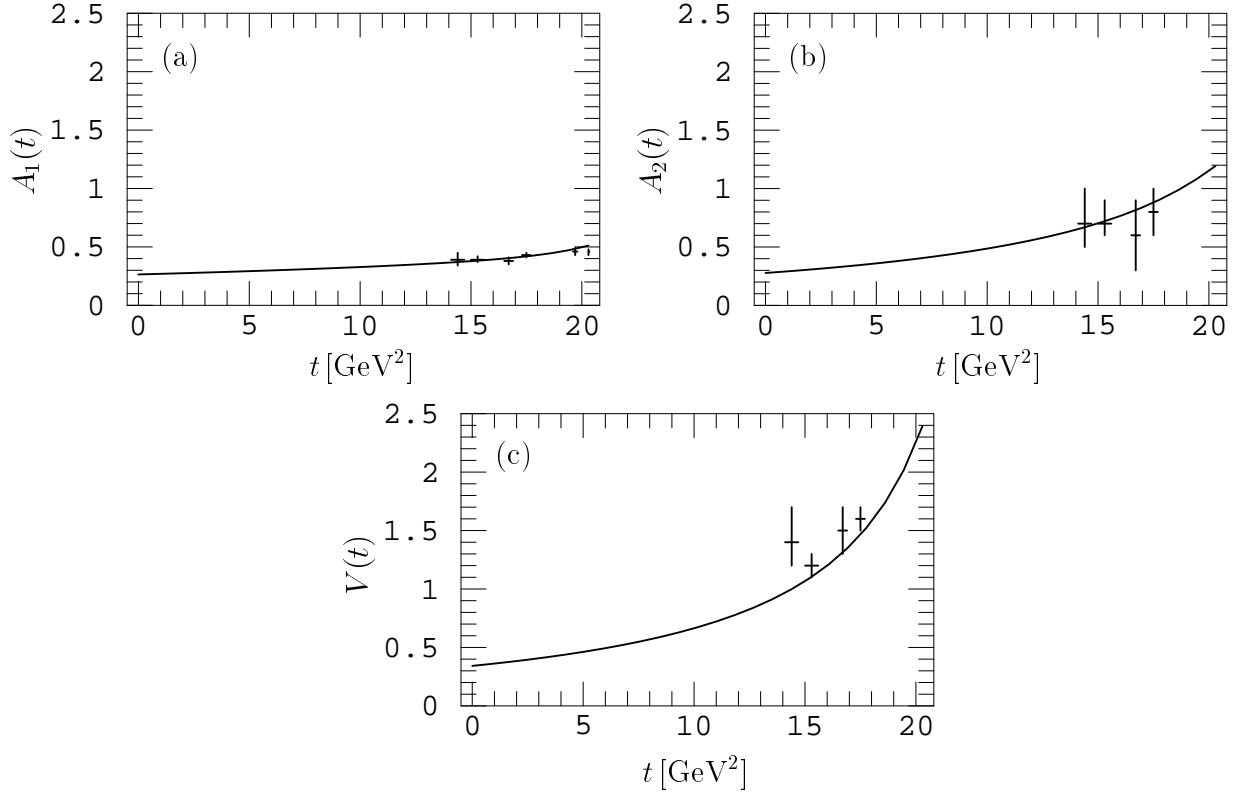


Figure 2: The semileptonic form factors as functions of  $t$  from light-cone sum rules [2] (solid lines) and lattice calculations (UKQCD) [9] (crosses).

reasonable ranges, in particular  $m_b = (4.8 \pm 0.2) \text{ GeV}$ :

$$\Delta A_1 = \pm 0.06, \quad \Delta A_2 = {}^{+0.09}_{-0.07}, \quad \Delta V = {}^{+0.10}_{-0.09},$$

nearly independent of  $t$ . For the integrated rates I obtain

$$B(B \rightarrow \rho e \nu) = (21.8 \pm 6.5) |V_{ub}|^2, \quad \Gamma_L/\Gamma_T = 0.52 \pm 0.10, \quad \Gamma_+/\Gamma_- = 0.016 \pm 0.003. \quad (4)$$

In Fig. 3 I also show the spectra  $d\Gamma/dE$  over the electron energy and  $d\Gamma/dt$ . It is now up to my experimental colleagues to provide better data that would allow to confirm or exclude the predictions from light-cone sum rules and lattice calculations.

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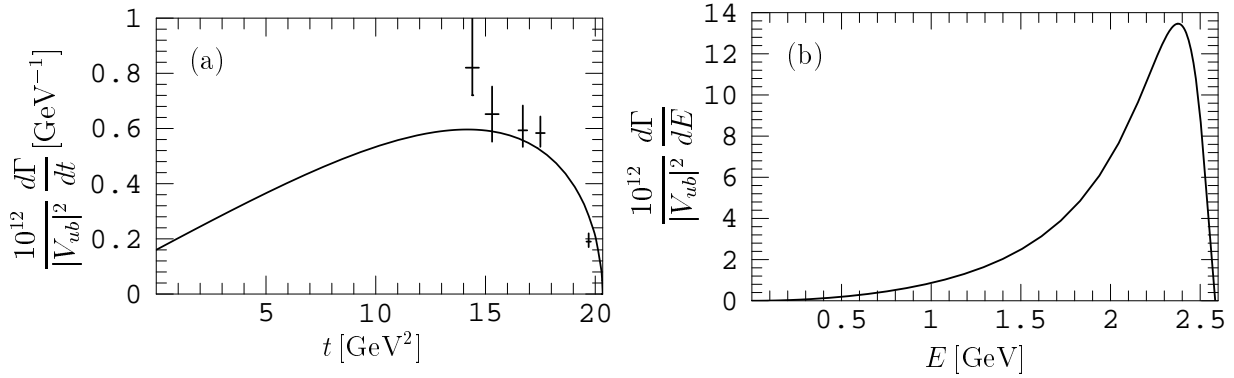


Figure 3: The spectrum  $d\Gamma/dt$  for the decay  $B \rightarrow \rho e \nu$  from light-cone sum rules (solid line) and lattice calculations (crosses). (b) The electron spectrum from light-cone sum rules (not available from lattice due to restricted information in  $t$ ).

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